

Welcome

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**Mathematics Is The Queen Of Science, And
Arithmetic The Queen Of Mathematics**

MATRICES - INTRODUCTION

Matrix algebra has at least two advantages:

- Reduces complicated systems of equations to simple expressions
- Adaptable to systematic method of mathematical treatment and well suited to computers

Definition:

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets

Examples:

3x3 matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix}$$

2x4 matrix

$$\begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix}$$

1x2 matrix



TYPES OF MATRICES

1. Column matrix or vector:

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

2. Row matrix or vector

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix}$$

3. Rectangular matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

4. Square matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

5. Diagonal matrix

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = 0$ for some or all $i \neq j$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Unit or Identity matrix - **I**

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} = 1$ for some or all $i = j$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. Null (zero) matrix – **0**: All elements in the matrix are zero

Matrices - Operations

ADDITION AND SUBTRACTION OF MATRICES

SCALAR MULTIPLICATION OF MATRICES

$$4 \times \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} \times 4 = \begin{bmatrix} 12 & -4 \\ 8 & 4 \\ 8 & -12 \\ 16 & 4 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

The number of columns of **A** must equal the number of rows of **B**

Example.

$$\begin{array}{ccccc} \mathbf{A} & \times & \mathbf{B} & = & \mathbf{C} \\ (1 \times 3) & & (3 \times 1) & & (1 \times 1) \end{array}$$

TRANSPOSE OF A MATRIX

$$A = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$$

MATRICES - OPERATIONS

INVERSE OF A MATRIX

DETERMINANT OF A MATRIX To compute the inverse of a matrix, the determinant is required

MINORS Each element in $\mathbf{A}$ has a minor

COFACTORS $C_{ij} = (-1)^{i+j} m_{ij}$

ADJOINT MATRICES The adjoint matrix of $\mathbf{A}$, denoted by $\text{adj } \mathbf{A}$, is the transpose of its cofactor matrix

$$\text{adj}A = C^T$$

Matrix Inversion

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

Practice time

Example

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

The determinant of $\mathbf{A}$ is

$$|\mathbf{A}| = (3)(-1-0) - (-1)(-2-0) + (1)(4-1) = -2$$

The elements of the cofactor matrix are

$$c_{11} = +(-1), \quad c_{12} = -(-2), \quad c_{13} = +(3),$$

$$c_{21} = -(-1), \quad c_{22} = +(-4), \quad c_{23} = -(7),$$

$$c_{31} = +(-1), \quad c_{32} = -(-2), \quad c_{33} = +(5),$$

The cofactor matrix is therefore

$$C = \begin{bmatrix} -1 & 2 & 3 \\ 1 & -4 & -7 \\ -1 & 2 & 5 \end{bmatrix}$$

$$\text{adj}A = C^T = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -4 & 2 \\ 3 & -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 2 & -4 & 2 \\ 3 & -7 & 5 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -1.0 & 2.0 & -1.0 \\ -1.5 & 3.5 & -2.5 \end{bmatrix}$$

Linear Equations

The equations may be expressed in the form

$$\mathbf{AX} = \mathbf{B}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Number of unknowns = number of equations = n



Thank You