Analysis of Continuum Spectra for The ^{209}Bi(n,d)^{208}Pb Reaction

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Abstract

Theoretical analyses of the continuum spectra for (n,d) reactions have been carried out at the incident energies of 37.5, 41.0, 45.0 and 49.0 MeV and at 40^0 angle. Direct reaction model has been used for the analysis of the spectrum. Experimental double differential cross sections for continuum spectra are predicted well by adopting an asymmetric Lorenzian form for the response function in the distorted wave Born approximation based cross-section calculations.

***Keywords*:** double differential cross section, (n,d), nuclear reactions, direct reaction model, DWBA analysis

1. Introduction

There have been a large number of investigations on the single-neutron pick up reactions on nuclei covering the entire periodic table of elements [1]. Some theoretical models have been proposed for the investigations of the (n,d) reactions [2-4], which cannot reproduce well the experimental data [5].

For the above reasons, to study the continuum spectra of the (p,d) reaction Syafarudin et al. [6] have given a procedure, which can give well prediction for the experimental spectra of the one-nucleon transfer reactions in higher emission energy region in the tens of MeV energy region, i.e., direct reaction region. However, the continuum spectrum in the direct reaction region is not possible to analyze so easily for the following reason is given below:

In the direct reaction, there should be emissions of separate line spectra because on hitting by the incident particle, the nucleon which goes to higher orbit falls to lower orbits including the orbit it was initially staying. This should never give a continuous spectrum. The problem is not totally solved. Therefore, to predict the direct reaction continuum spectra of the (p,d) reaction, Matoba group adopted an approach suggested by Lewis [7]. By adopting the approach in several studies on the direct reaction scheme, Matoba et al. reached eventually to a decision to solve this critical problem [8, 9]. First, this method has been successfully applied for the (p,d) reaction on ^{48}Ca and ^{58,60,62,64}Ni at 65.0 MeV [6], ^{96}Mo at 50.0 MeV [1], ^{100}Mo at 21.0 and 50.0 MeV [10, 11], ^{58}Ni, ^{90}Zr, ^{197}Au and ^{209}Bi at 42.0 and 68.0 MeV [12], ^{197}Au(p,d)^{196}Au reaction at 68.0 MeV [13], ^{27}Al(p,d)^{26}Al reaction at 42.0 MeV [14]. The present work for the (n,d) reactions is an advanced form of the (p,d) reaction analyses and have been applied for ^{nat}Fe at 75.0 MeV [12], ^{209}Bi at 42.0, 45.0, 49.0, 53.5 and 62.7 MeV for 20^0 angle [15], ^{209}Bi at 41.0 – 53.5 MeV for 50^0 angle [16], ^{209}Bi at 28.5, 34.5, 37.5, 41.0, 45.0, 49.0, 53.5 and 62.7 MeV at laboratory angle 30^0 [17, 18].

There are many applications for fast neutrons. The main application fields of the fast neutrons are accelerator-driven sub-critical systems (ADS) and bismuth (Bi) is also the target nucleus in the ADS reactor systems [19]. This work is a part of our systematic works and concerned with the ^{209}Bi(n,d)^{208}Pb reaction at the incident energies of 34.5, 37.5, 41.0, 45.0 and 49.0 MeV for 40^0 angle. The theoretical spectra are reproduced well as compared with experimental data in the direct reaction region. Details of the experimental procedure and the results have been reported in ref. [20].

2. Materials and Methods

Theoretical Calculations

Double differential cross sections are analyzed with the direct reaction model composed by an incoherent sum of the DWBA predictions from all constituent shell-orbits, as expressed by

$$\frac{d^2\sigma}{d\Omega dE} = 2.30\sum_{l,j}\left[\frac{C^2 s_{l,j}(E)}{2j+1}\times\left(\frac{d\sigma}{d\Omega}\bigg|_{l,j}^{DW}(E)\right)\right] \qquad (1)$$

and the spectroscopic factor is as

$$C^2S_{l,j}(E) = \left(\sum C^2 S_{l,j}\right)\times f_{l,j}(E). \qquad (2)$$

where $d\sigma/d\Omega\big|_{l,j}^{DW}(E)$ is the cross-section calculated by the DWBA code, DWUCK-4 [21] and $C^2S_{l,j}(E)$ is the spectroscopic factors of all possible states.

The sum rule of the spectroscopic factors of nucleon orbits for $\mathrm{T}\pm 1/2$ isospin states are estimated with a simple shell model prescription [22] which is as follows

$$\sum c^2 s_{l,j} = \begin{cases} n_n(l,j) - \dfrac{n_p(l,j)}{2T+1} & for\ \mathrm{T}_< = \mathrm{T} - \dfrac{1}{2} \\ \dfrac{n_p(l,j)}{2T+1} & for\ \mathrm{T}_> = \mathrm{T} + \dfrac{1}{2} \end{cases} \qquad (3)$$

Here $n_n(l, j)$ and $n_p(l, j)$ are the numbers of neutrons and protons respectively for each l, j orbit and T is the target isospin.

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This sum rule of each orbit is true for the (p,d) reaction, but for the (n,d) reaction, we consider no contribution for $n_n(l, j)$ i.e., no contribution for isobaric analogue state (IAS) in the spectrum. So we apply 100% contribution for the spectra only for $n_p(l, j)$.

$$C^2 S_{l,j} = \frac{n_{p(l,j)}}{2T+1} \quad (4)$$

The distribution of strength function over the spectra is predicted by using an asymmetric Lorentzian function [8-9, 23-24] as follows

$$f_{l,j}(E) = \frac{n_0}{2\pi} \frac{\Gamma(E)}{\left(|E - E_F| - E_{l,j}\right)^2 + \Gamma^2(E)/4}, \quad (5)$$

and

$$\int_0^{\alpha} f_{l,j}(E) dE = 1 \quad (6)$$

where n_0 is the renormalization constant and E_F is the Fermi energy. The Fermi energy can be calculated by using an empirical formula [25]. The sum rule of spectroscopic factors and the centroid energies ($E_{l,j}$) calculation for $J = l \pm 1/2$ shell orbits have been done by using a BCS calculation. In this calculation, we need single particle energies to calculate the centroid energy, where single particle energies are calculated by the prescription of Bohr and Mottelson [26].

The spreading width (Γ) is expressed by a function proposed by Brown and Rho [27] as well as by Mahaux and Sartor [23] as

$$\Gamma(E) = \frac{\varepsilon_0 (E - E_F)^2}{(E - E_F)^2 + E_0^2} + \frac{\varepsilon_1 (E - E_F)^2}{(E - E_F)^2 + E_1^2}, \quad (7)$$

where ε_0, ε_1, E_0 and E_1 are constants which express the effects of nuclear damping in the nucleus [8]. The estimated parameters [8] are

$$\varepsilon_0 = 19.4 \text{ MeV}, \quad E_0 = 18.4 \text{ MeV} \quad (8)$$

$$\varepsilon_1 = 1.40 \text{ MeV}, \quad E_1 = 1.60 \text{ MeV}$$

3. Results and Discussion

Theoretical calculations for the DDXs were done of the $^{209}Bi(n,d)^{208}Pb$ reaction by adopting an asymmetric Lorenzian form for the response function in the distorted wave Born approximation based cross-section calculations. Here the incident energies are 34.5, 37.5, 41.0, 45.0 and 49.0 MeV for 40^0 angle. No theoretical data for the $^{209}Bi(n,d)^{208}Pb$ reaction are available in the region of tens of MeV to compare with our theoretical calculated data.

The experimental and theoretical results are given by the circles and lines, respectively. Three global potentials were used here for neutrons, while for deuteron an adiabatic potential based on the proton and neutron potentials [28-30] were constructed for the DWUCK-4 calculations as shown in Table 1. The dotted, short-long-dashed and solid lines represent the DDX for Koning and Delaroche [28], Menet *et al*. [29] and Becchetti and Greenlees [30] potentials, respectively.

Table 1: Optical model parameters used in the DWBA calculations for the $^{209}Bi(n,d)^{208}Pb$ reactions

Koning and Delaroche potential (27)

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Neutron	a	1.24	0.65		a	a	1.25	0.51	a	1.08	0.59
Deuteron	b	1.24	0.65	1.22	b	b	1.25	0.57	b	1.08	0.59
Proton	c	1.25	0.65	1.22							

[a]Neutron potentials [27]
[b]Adialbatic potentials [27]
[c]Well depth adjusted to fit the separation energy

Menet potential [28]

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Neutron	a	1.16	0.75		b	b	1.37	c	6.04	1.06	0.78
Deuteron	d	1.16	0.75	1.25	e		1.37	f	6.04	1.06	0.78
Proton	g	1.25	0.65	1.25							

[a]$V = [49.9 - 0.22E_n - 26.4(N-Z)/A]$ MeV
[b]$W_v = [1.2+0.09\ E_n]$ MeV, [b]$W_s = [4.2 - 0.05\ E_n+15.5(N - Z)/A]$ MeV, E_n is the neutron kinetic energy
[c]$a' = 0.74 - 0.008\ E_n$(fm)
[d]$V = [99.8 - 0.44(E_n/2) + 0.4Z/A^{1/3}]$ MeV
[e]$W_v = [2.4+0.18\ (E_d/2)]$ MeV, [e]$W_s = [8.40 - 0.10\ (E_d/2)]$ MeV, E_d is the deuteron kinetic energy
[f]$a' = 0.74 - 0.008\ (E_d/2) + 1.0\ (N - Z)2A$ (fm)
[g]Well depth adjusted to fit the separation energy

Becchetti and Greenlees potential [29]

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Neutron	a	1.17	0.75		b	b	1.26	0.58	6.20	1.1	0.75
Deuteron	c	1.17	0.78	1.25	d	d	1.29	0.65	6.20	1.06	0.75
Proton	e	1.25	0.65	1.25							

[a] $V = [56.3 - 0.32\ E_n - 24\ (N-Z)/A]$ MeV
[b] $W_v = [0.22E_n - 1.56]$ MeV, $Ws = [13.0 - 0.25\ E_n - 12(N-Z)/A]$ MeV, E_n is the neutron kinetic energy
[c] $V = [110.3 - 0.64(E_d/2)+0.4Z/A^{1/3}]$ MeV
[d] $W_v = [0.44\ (E_d/2) - 4.26]$ MeV, $Ws = [24.8 - 0.50\ (E_d/2)]$ MeV, E_d is the deuteron kinetic energy
[e] Well depth adjusted to fit the separation energy

	Nonlocality parameter (fm)	Finite-range parameter (fm)	spin-orbit term λ
Proton	0.85	0.621	25
Deuteron	0.85	0.621	25
Neutron	054		25

From Figs.1- 4 we can see that the theoretical results are in good agreement with the experimental ones for the incident energies of 37.5, 41.0 and 45.0, while for the incident, the theoretical result is somehow a bit underestimated. Therefore, it is seen that the continuous spectrum in the low energy region of the emitted deuteron remains a problem, thus far not clearly understood. Further studies are needed on this aspect of the nuclear reaction.

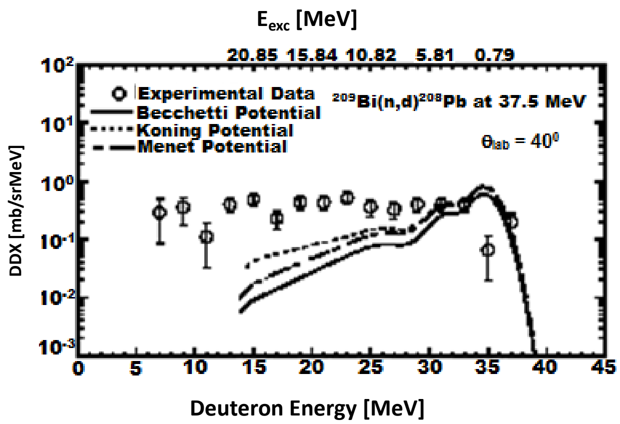

Fig. 1: Double differential cross sections for $^{209}Bi(n,d)^{208}Pb$ reaction at 40^0 laboratory angle for 37.5 MeV incident energy

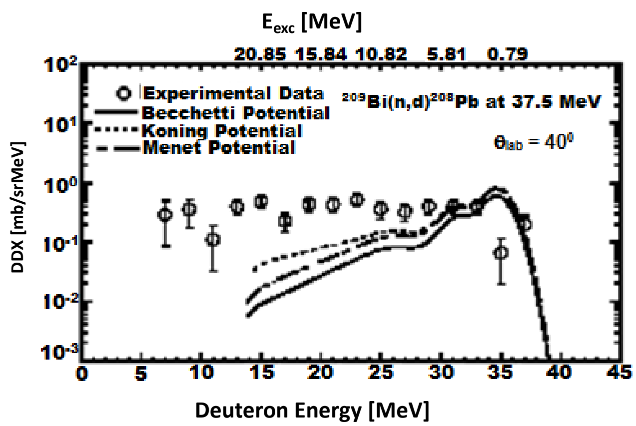

Fig. 2: Double differential cross sections for $^{209}Bi(n,d)^{208}Pb$ reaction at 40^0 laboratory angle for 41.0 MeV incident energy

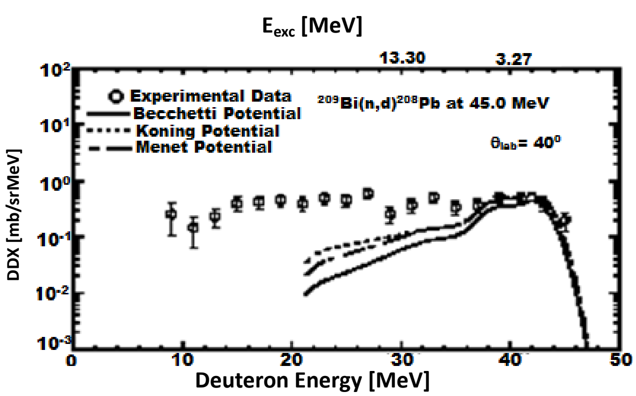

Fig. 3: Double differential cross sections for $^{209}Bi(n,d)^{208}Pb$ reaction at 40^0 laboratory angle for 45.0 MeV incident energy

In spite of the above fact at the incident energy of 49.0 MeV, it shows that the overall agreement between the calculated spectra and experimental ones is good at all energies both in shape and magnitude. The calculated double differential cross sections agree with experimental data only above tens of MeV energy region, because our calculated energy

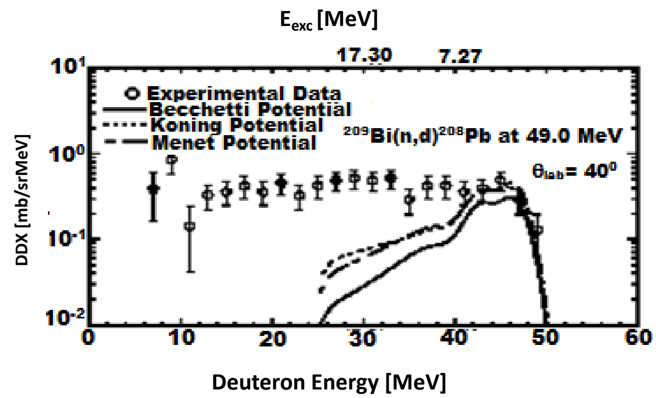

Fig. 4: Double differential cross sections for $^{209}Bi(n,d)^{208}Pb$ reaction at 40^0 laboratory angle for 49.0 MeV incident energy

spectrum regions are treated in direct reaction scheme. The direct reaction scheme is actually applicable to higher energy ranges, while the compound nuclear model strictly applied to low energy region.

4. Conclusion

The $^{209}Bi(n,d)^{208}Pb$ reaction has been studied here with a wide range of incident energies. The incident energy range is 34.5 –49.0 MeV and for 40^0 laboratory angle. Direct reaction model has been used for the analysis of the spectrum. Quite satisfactory fits to the DDX for the above reactions have been obtained by our method of calculation in the direct reaction region. The shapes of double differential cross-sections affirmatively agree with the experimental ones. The calculation of theoretical value of spectra without using any arbitrary renormalization of the cross-sections made our theoretical method more applicable for the data analysis.

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